Unit 5: Summation Notation (Sigma Notation)

Name: _____

Sigma = $\sum$ (Greek letter) used for sum of a sequence

$$\overset{\text{upper}}{\underset{\text{lower}}{\sum}} \text{ expression}$$

Examples:

$\sum_{k=1}^{5} k = 1+2+3+4+5 = 15$

$\sum_{k=2}^{6} 3k = 3(2)+3(3)+3(4)+3(5)+3(6) = 60$

$\sum_{k=1}^{5} k^2 = 1^2+2^2+3^2+4^2+5^2 = 55$

$\sum_{k=0}^{3} k^3 = 0^3+1^3+2^3+3^3 = 36$

Properties of Sigma Notation

1. $\sum_{k=1}^{n} ca_k = c\sum_{k=1}^{n} a_k$

2. $\sum_{k=1}^{n} (a_k \pm b_k) = \sum_{k=1}^{n} a_k \pm \sum_{k=1}^{n} b_k$

Ex: $\sum_{k=1}^{3} 4k = 4\sum_{k=1}^{3} k$

(SUM) Ex: $\sum_{k=1}^{3} (k^2+k) = \sum_{k=1}^{3} k^2 + \sum_{k=1}^{3} k$

(DIFFERENCE) Ex: $\sum_{k=1}^{3} (k^3-k^2) = \sum_{k=1}^{3} k^3 - \sum_{k=1}^{3} k^2$

Special Summation Formulas

1. $\sum_{k=1}^{n} c = nc$

2. $\sum_{k=1}^{n} k = 1+2+3+\ldots+n = \frac{n(n+1)}{2}$

3. $\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$

4. $\sum_{k=1}^{n} k^3 = 1^3+2^3+3^3+\ldots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Calculator Shortcut

sum(seq(expression, variable, lower,upper)

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Examples:

Write out the terms of the given expressions and then find each sum.

1. $\sum_{k=1}^{5}(-1)^k\left(k^2+3\right)=$

2. $\sum_{k=0}^{4}\left(\frac{1}{k^2+1}\right)=$

Write the following sums using sigma notation.

3. $2+4+6+8+10+12+\ldots+44$

4. $\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\frac{1}{81}+\frac{1}{243}$

5. $f(x_1)+f(x_2)+\cdots+f(x_n)$

Using your properties and the summation formulas evaluate the following sum.

6. $\sum_{k=1}^{50}\left(2k^2+3k+4\right)=$

7. $\sum_{k=2}^{10}\left(k^3+k^2\right)=$

8. $\sum_{k=11}^{20}\left(-k^2-5k\right)=$

Now, use your calculators to confirm your answer found above.

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Unit 5 Worksheet 5
AP Calculus AB

Write out the terms of the given expressions and then find each sum.

1. $\sum_{k=1}^{5}(3k-1)$
2. $\sum_{i=1}^{6} 2i^2$
3. $\sum_{i=3}^{5} \frac{2}{1+i}$
4. $\sum_{j=2}^{6} (j+1)^2$
5. $\sum_{i=1}^{5} (-1)^i 2^{i-1}$
6. $\sum_{i=2}^{4} \frac{(-1)^i}{i(2i+1)}$
7. $\sum_{k=1}^{6} \sin\left(\frac{k\pi}{2}\right)$
8. $\sum_{k=1}^{7} \cos(k\pi)$

Write the following sums using sigma notation.

9. $1+2+3+\cdots+98$
10. $2+4+6+\cdots+100$
11. $1+4+9+16+25+36$
12. $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots-\frac{1}{50}$
13. $\frac{3}{5}+\frac{3}{25}+\frac{3}{125}+\frac{3}{625}$
14. $a_1+a_2+a_3+\cdots+a_n$
15. $b_3+b_4+b_5+\cdots+b_{22}$
16. $f(c_1)+f(c_2)+\cdots+f(c_n)$
17. $f(w_1)\Delta x+f(w_2)\Delta x+\cdots+f(w_n)\Delta x$
18. $-1+\frac{1}{8}-\frac{1}{27}+\frac{1}{64}-\frac{1}{125}$

Calculate each of the following if $\sum_{i=1}^{10} a_i = 40$ **and** $\sum_{i=1}^{10} b_i = 50$**.**

19. $\sum_{i=1}^{10} (2a_i + b_i)$
20. $\sum_{i=1}^{10} (3a_i - 2b_i)$
21. $\sum_{i=1}^{10} (4a_i - b_i + 2)$
22. $\sum_{i=1}^{10} (2b_i + 4)$

Using your properties and the summation formulas, evaluate the following sum. Then, verify the accuracy of your answer using your calculator shortcut.

23. $\sum_{k=1}^{8} (3k^2 + 7k - 10) =$
24. $\sum_{k=1}^{10} 5k^2(k+4)$
25. $\sum_{k=1}^{45} (k^3 - k^2) =$
26. $\sum_{k=2}^{64} (8k + 7k^2) =$
27. $\sum_{k=17}^{40} (-k^2 + k) =$
28. $\sum_{i=1}^{n} (2i-3)^2$